Two-dimensional inclined chute flows: Transverse motion and segregation

Gaelle Berton, Renaud Delannay, Patrick Richard, Nicolas Taberlet, and Alexandre Valance
Groupe Matière Condensée et Matériaux, UMR 6626, Université Rennes 1, F35042 Rennes Cedex, France
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We present an experimental study of two-dimensional dense inclined chute flows consisting of both monodisperse and bidisperse disks. We analyzed the trajectories of the particles within the flow in a steady regime. (i) In monodisperse flows, particles are arranged in layers that are in motion relative to one another, and it is found that the particles have a nonzero probability of being transferred to adjacent layers. We measured the mean time spent by a particle in a given layer. This residence time is found to decrease with increasing layer height. The particle transfer between layers can be interpreted as transverse motion of a diffusive nature. The diffusion coefficient associated with each layer increases linearly with the layer height. (ii) In polydisperse flows consisting of a small percentage (less than 1%) of small disks among large ones, the small particles have a net downward motion on which a fluctuating behavior is superimposed. At short times, the small particle motion can be described as a biased Brownian motion. The ratio of the characteristic time of diffusion to that of convection is found to increase with the layer height, indicating that the segregation process is more efficient in the upper layers of the flow. At longer times, the transverse motion of the small particles seems to differ greatly from a classical biased Brownian motion.

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I. INTRODUCTION

Granular material flows on inclined chutes still pose some challenging problems related to modeling. Unlike classical fluids, which are correctly modeled by Navier-Stokes equations, there is no complete and general theory for the description of granular fluids. It is therefore crucial to better understand the physical mechanisms which drive granular flows, especially since they occur in geophysical contexts (rock avalanches, landslides) [1,2] and in industrial applications.

According to the nature of the flow regime (frictional or collisional), several formulations have been suggested. If the granular medium is dilute and highly sheared, the particles interact collisionally, which dissipates a part of their kinetic energy. In this case, constitutive laws may be deduced from a microstructural approach similar to kinetic theory for dense gases [3]. On the other hand, if the granular medium is dense and slowly sheared, the particles have persistent contacts and dissipate energy by friction. The forces between particles have static origin and the constitutive law is plasticlike.

Inclined granular chute flows generally do not fall into these two limit regimes. They belong to an intermediate regime where both friction between particles and collision play important roles. In this regime, no sound convincing constitutive law has been proposed. Despite the numerous experimental [4–10], numerical [11,12], and theoretical [13–15] works devoted to granular chute flows, their description and prediction are still a challenge.

Most experimental studies have concerned three-dimensional (3D) flows, where it is nearly impossible to determine the particle velocity because inserting a probe seriously disturbs the measurement. To avoid the experimental difficulties encountered in the case of 3D flows, Drake [6] proposed to use a two-dimensional geometry where particles are confined between two glass walls. This geometry is well suited to a detailed analysis of the microstructure and kinematics of the flow via a high speed camera. The purpose of the present article is to analyze in a 2D inclined granular chute flow the process of particle layer transfer. This study is motivated by the understanding and modeling of the segregation phenomenon in granular chute flows. When particles of the same density but different sizes are present in a flow, the large particles migrate toward the top free surface whereas the small particles move downward to reach the bottom surface. This migration process is intimately related to the mechanism driving the particle transfer between adjacent layers. As a first step, it is important to analyze this layer transfer process carefully even in the case of monodisperse flows. In that case, the probability for a particle to be transferred from one layer to another is not reduced to zero. What are the characteristics of the layer transfer process? What is the driving mechanism of this process? These simple issues have not received much attention among granular physicists and only a little is known [29].

The present article will therefore deal, in the first part, with monodisperse flows, while the segregation process itself is treated in the second part. In the first part, we focus on the transverse motion of particles in a fully developed monodisperse flow in a 2D rough inclined chute. We report the following results: (i) although the flow is organized in layers that are in motion relative to one another, particles from a given layer have a nonzero probability of being transferred into adjacent layers; (ii) the mean transverse displacement of particles from the bulk flow shows a surprising mean upward drift; and (iii) the time spent by an individual particle in a given layer is found to be dependent on the layer considered (it is shorter for a particle in an upper layer than for one in a lower layer). This particle transfer between layers can be interpreted as transverse motion of a diffusive nature. From the experimental data, we estimated, in particular, the diffusion coefficient associated with each layer, which is found to increase linearly with the layer height. A natural consequence of the variation of the diffusion coefficient with the height is to induce a mean upward motion of the particles.

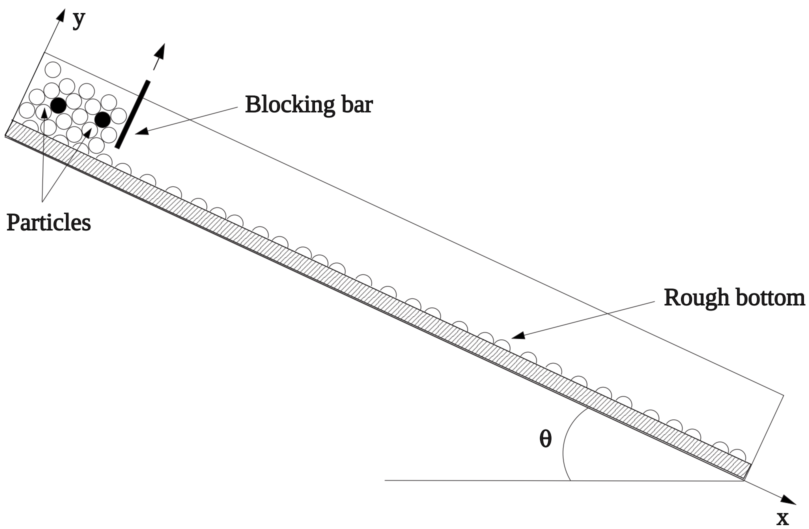

FIG. 1. Experimental setup. The higher part of the channel is used as a reservoir of particles; the flow is triggered by removing the blocking bar.

In the second part, we focus on the segregation process within bidisperse flows consisting of dilute mixtures (where the proportion of small disks among large ones is less than 1%). We analyzed the downward transverse motion of the small particles, which is a manifestation of the segregation process. We found that the small particles have, on average, a net downward transverse motion, on which a fluctuating movement is superimposed. The particle motion can be described at short times by a biased Brownian motion. The diffusive process therefore competes with the convective motion. We estimated the respective strengths of both processes through the Péclet number, defined as the ratio of the characteristic time of diffusion to that of convection. This ratio is found to be of the order of unity and increases with the layer height. This means that the segregation process is more efficient in the upper layers of the flow. At longer times, the transverse motion of the small particles seems to differ greatly from a classical biased Brownian motion and bears some resemblance to a Lévy flight.

The paper is organized as follows. Section II is devoted to the monodisperse flow experiments. We first describe the experimental setup used for the chute and present the general features of the monodisperse flow in terms of velocity and granular temperature. We then analyze in detail the transverse motion of the particles and discuss the experimental outcomes. Section III is devoted to the bidisperse flow experiments. We first report the characteristics of the bidisperse flow in terms of velocity profiles for both types of particles. We then present the experimental outcomes for the segregation process through the transverse motion of the small particles and discuss the nature of this transverse motion. Finally, a general conclusion and outlook are given in Sec. IV.

II. MONODISPERSE FLOWS

A. Experimental setup

The chute flow experiments have been performed using the setup sketched in Fig. 1. Unlike most granular chute experiments, the particles used here are not spheres but disks. Although the use of disks instead of spheres enhances the friction with the sidewall, it allows us to work with monosized assemblies as well as with binary mixtures. In the monodisperse flow experiments, we use polystyrene disks of $d = 8$ mm diameter and 3 mm thickness.

The inclined chute is 2 m long, and the flowing particles are confined between two smooth glass sidewalls. The gap between the two walls is 3.3 mm wide (slightly wider than the particle thickness). The presence of sidewalls introduces an additional friction force on the disk particles, which amounts to reducing the gravity force. The bottom is made rough by randomly placing half particles. More precisely, the distance between the centers of two successive half particles is chosen in the interval $[d, \sqrt{3}\, d]$ according to a uniform random distribution. ($\sqrt{3}d$ is the critical interparticle distance corresponding to the situation where a particle from the flow can fill the void left between the bottom particles.) The higher part of the channel is used as a reservoir and filled with 1260 particles which are blocked by a bar. To trigger the chute, the bar is removed. At the end of the chute, the particles are free to flow out from the channel.

We estimated, with this setup, the critical angle θ_c where $h_{stop}(\theta)$ vanishes. $h_{stop}(\theta)$ is defined as the minimum thickness of the flowing layer necessary to observe a uniform flow at a given inclination θ [10]. We find that $\theta_c \approx 32°$ within a few degrees. All experiments were performed with an inclination θ of 36°, greater than θ_c. It should be noted that, due to wall friction, steady uniform flows exist for angles much greater than θ_c.

Flow experiments are recorded by a high speed video camera (fastcam photron) at a rate of 500 images per second and a resolution of 512×240 pixels. This frequency is high enough to track the motion of each grain of the flow. Image processing software computes the position of the center of mass of the disks on each image, therefore allowing one to extract the trajectories and calculate the kinematic properties of the particles.

During the discharge of the reservoir, which lasts a few seconds, we observe a fully developed shear flow over a window 20 cm long, 1 m downstream of the reservoir. The fully developed shear flow is characterized by a constant

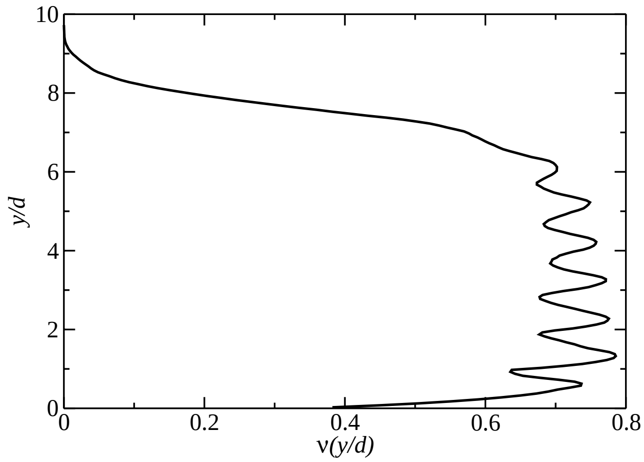

FIG. 2. Packing fraction profile of the flow. A stratification is developing within the flow.

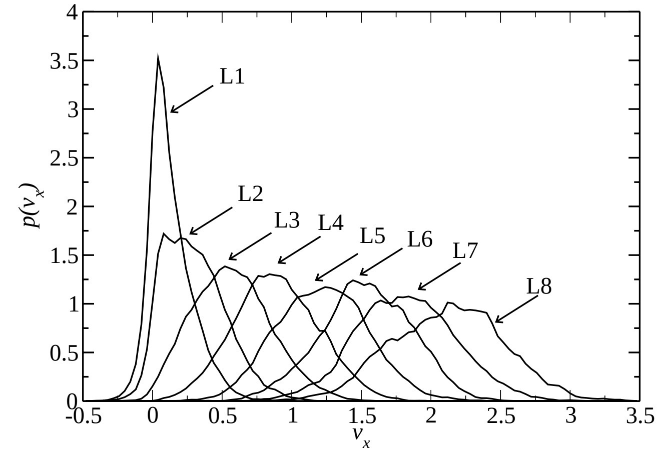

FIG. 4. Velocity distribution in the different layers of the flow.

height equal to $8d$. The flow is dense: the packing fraction is rather constant in the bulk flow and is about 0.7 (see Fig. 2). A constant packing fraction is also observed in dense flows of spherical particles in 2D and 3D [6–8]. Within the flow, a stratification develops: layers of one particle width are built up and slide over each other. Close to the top free surface, the layers are less structured, the medium is more fluidized, and therefore the packing fraction is lower, as can be seen in Fig. 2.

B. Velocity and temperature profiles

We will define the *Ox* axis to be parallel to the flow direction and the *Oy* axis perpendicular to it. The velocity measurements are presented in Fig. 3. The modulus of the average dimensionless velocity and its components along the *x* and *y* directions are plotted as functions of the distance from the base of the flow. The ‘‘instantaneous’’ velocity of the particles is calculated from two successive positions taken at a sampling rate of 500 Hz and the averages have been performed over time (i.e., over the video recording duration) and space (i.e., over the *x* direction within the observation window). The velocity of the particles increases nearly linearly with the distance from the base. The shear rate here is about 10 s^{-1}. The linearity of the velocity profile is a classical result in granular dense flows consisting of spherical particles in 3D and 2D [6–8]. In the transverse direction, the mean velocity of the particles is found to be equal to zero within the experimental uncertainties.

We have also plotted in Fig. 4 the distributions of V_x in the different layers of the flow. We have labeled each layer of the flow by Li, where the index $i=1$ refers to the bottom layer and $i=8$ to the top layer. The width of the distribution (which is nothing but a measure of the ‘‘granular temperature,’’ as we shall see later on) increases as one moves away from the bottom layer. These velocity distribution are very close to Gaussian distributions except for layers close to the bottom (L1, L2, and L3). For these three layers the distributions are no longer symmetric and their peaks are sharper than a Gaussian one.

We have plotted in Fig. 5 the variance of the velocity distributions as a function of the distance *y* from the base of the flow in dimensionless variables. The variance of the ve-

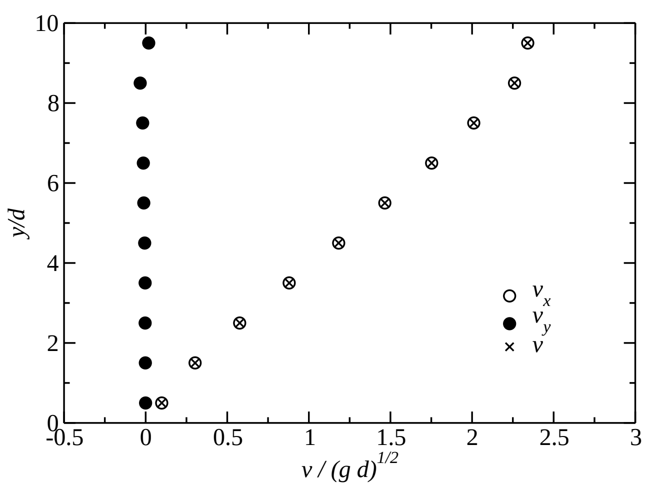

FIG. 3. Velocity profile: (×) velocity V, (○) V_x, and (●) V_y.

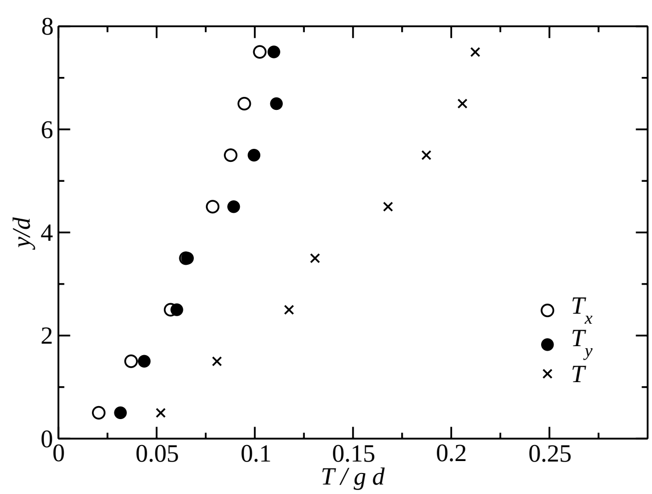

FIG. 5. Granular temperature profile: (○) $T_x=\overline{V_x^2}-\overline{V_x}^2$, (●) $T_y=\overline{V_y^2}-\overline{V_y}^2$, and (×) $T=T_x+T_y$.

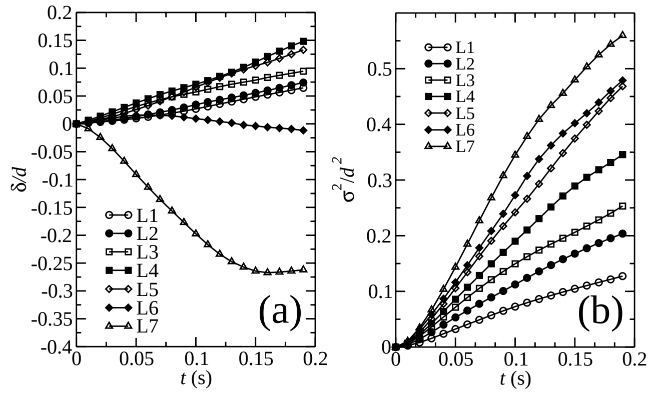

FIG. 6. Experimental measurements: (a) mean vertical displacements as a function of time for different initial positions of particles; (b) variance of vertical displacements as a function of time for different initial positions of particles.

locity distribution is a measure of the so-called granular temperature $T_x \sim \overline{V_x^2} - \overline{V_x}^2$, $T_y \sim \overline{V_y^2} - \overline{V_y}^2$, and $T = T_x + T_y$. One should point out that the concept of "granular temperature" has a clear meaning when the granular flow is collisional, since it then corresponds to the standard definition of the kinetic temperature calculated from the rms velocity measured during free flights. For dense flows where most of particles have lasting contacts, this is no longer the case since velocities are measured during contacts. However, in the literature on granular media, one still uses the term "granular temperature" for the rms velocity calculated in this way.

The granular temperature is close to zero at the base of the flow and increases with the height y. For low y, T scales as $T \sim y^{\alpha}$ with $\alpha \approx 0.65$. One can note in addition that the temperatures along the x and y directions are of the same magnitude: there is no anisotropy. The measured temperature profile differs from those found in 2D and 3D simulations of dense flows of spheres where the temperature is constant in the bulk flow [8,16]. The origin of this difference is not yet well understood but it may be due to an anomalously highly dissipative bottom.

C. Transverse displacements

We focus here on the transverse motion of the particles according to their height in the flow.

In Fig. 6(a), we have plotted the mean transverse displacement of the flow particles according to their initial transverse position in the flow. The mean transverse displacement $\delta_i(t)$ for particles initially located in the layer Li has been calculated as follows:

$$\delta_i(t) = \frac{1}{N_i} \sum_{j=1}^{N_i} [y_j(t) - y_j(0)], \tag{1}$$

where N_i is the number of recorded particles initially in the layer Li. $y_j(0)$ corresponds to the initial transverse position of the particle j and $y_j(t)$ to its position after an elapsed time t. We estimated $\delta_i(t)$ for $i = 1,2,\ldots,7$ and for $0 \leqslant t \leqslant 0.2$ s. For the upper layers and longer times, our statistics are too weak. We can see in Fig. 6(a) that the mean lateral displacement is not reduced to zero and depends on the particle position in the flow. Particles from the lower layers (i.e., $i = 1,2,\ldots,5$) migrate upward, whereas the ones from the upper layers have a mean downward motion. The maximum positive drift velocity is approximately 0.75 cm/s and corresponds to the mean displacement of the particles of layer L4.

An important remark should be made. The existence of a mean upward or downward motion for the particles is not in contradiction with the fact that the mean transverse velocity of the flow (or mean transverse mass flux) is zero. The mean local speed of the particles in each layer is zero, but, as we see next, the existence of a gradient in the mass diffusion coefficient perpendicular to the flow can partly explain the nonzero mean displacement of the particles once they have visited several layers.

We have also evaluated [see Fig. 6(b)] the variances of the vertical displacements, defined by

$$\sigma_i^2(t) = \frac{1}{N_i} \sum_{j=1}^{N_i} [y_j(t) - y_j(0)]^2 - \delta_i^2(t). \tag{2}$$

The variances of the vertical displacements are found to increase linearly with time for short times. Furthermore, the higher the position of the particle in the flow, the greater the fluctuations. The interesting outcome is the linear evolution of the variance with the time. This suggests the existence of a diffusive behavior for short times. An effective diffusion coefficient can be evaluated from the slope of the curve $\sigma^2(t)$ for short times ($\sigma^2 \simeq 2Dt$). It is found that D increases linearly with the transverse position y in the flow (see Fig. 8 below):

$$2D(y) = a\frac{y}{d} + b, \tag{3}$$

with $a \approx 0.296$ cm^2/s and $b \approx 0.312$ cm^2/s.

We also measured the average time spent by the particles within a given layer Li before visiting another one. The evaluation of this residence time is slightly underestimated due to the fact that some particles stay in a given layer longer than the recording time. Figure 7 gives a representation of the mean residence time τ_r according to the layer considered. When particles approach the free surface of the flow, the residence time decreases. These results completely support the conclusion drawn from the transverse displacement fluctuations.

It is also possible to derive an effective diffusion coefficient from the measured residence time τ_r. A classical calculation based on a ruin problem (or random walk [17]) allows one to link the residence time to the diffusion coefficient D associated with the diffusion process:

$$2D = \frac{d^2}{4\tau_r}. \tag{4}$$

In Fig. 8, we have presented the diffusion coefficient evaluated directly from the fluctuations of the transverse displace-

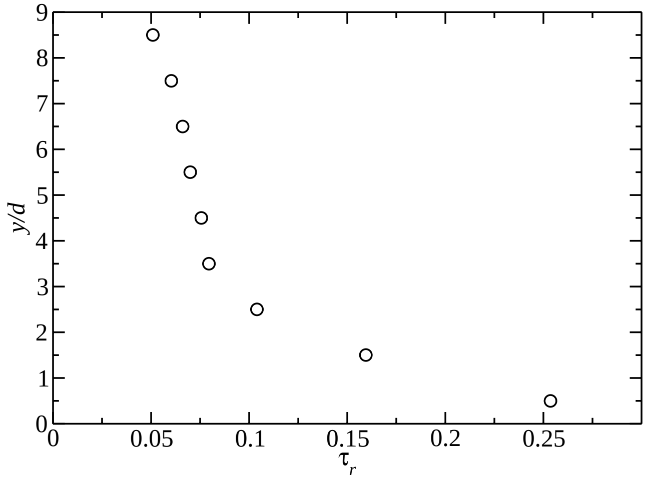

FIG. 7. Evaluation of the residence time in each layer.

ments and that calculated from the residence time [Eq. (4)]. Both data sets show quite similar results. This strongly reinforces the hypothesis of a transverse diffusion process.

D. Discussion

We would like to discuss here the consequences of the height-dependent diffusion coefficient for the transverse motion of the particles. In particular, we want to check whether the inhomogeneity of the diffusion coefficient throughout the flow can explain not only qualitatively but also quantitatively the nonzero net transverse motion of the particles of the flow. The issue is thus the following: Given a particle diffusing within a bounded layered medium where the diffusion coefficient varies from one layer to another, what is its average displacement at short times and what is the effect of the boundaries? The simplest and quickest way to answer these questions is to carry out simulations based on a classical

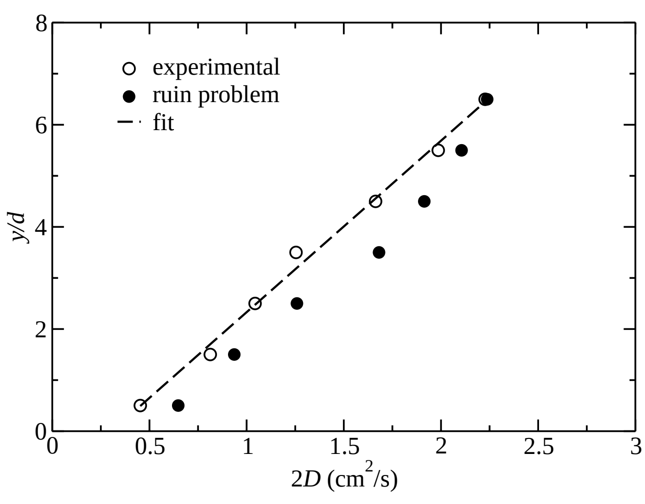

FIG. 8. Diffusion coefficient as a function of the height from the base of the flow: (○) values obtained from the measurement of the lateral displacement fluctuations, (●) values derived from the experimental evaluation of the residence time, and (dashed line) linear regression.

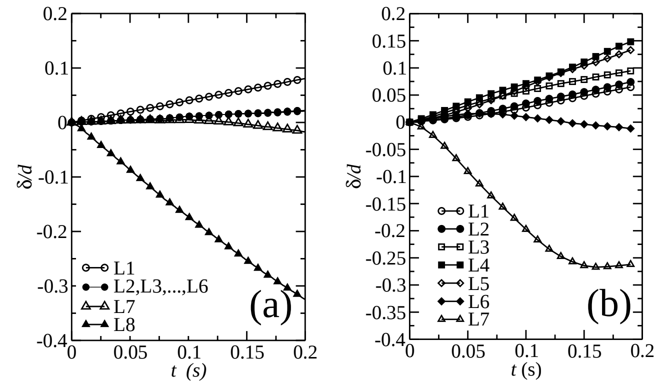

FIG. 9. Mean vertical displacements as a function of time for different initial particle positions: (a) results of simulations based on random walk; (b) experimental results.

random walk, which is currently used in fluid mechanics to study dispersion and diffusion problems [18].

We have therefore simulated the transverse displacement of a single particle by a one-dimensional random walk along the y direction. The particle moves in a layered medium where the diffusion coefficient D_i of the layer Li is taken to increase linearly with the layer height, as suggested by the experimental results. The medium comprises eight layers and is bounded by two reflective walls, which mimic the presence of the rough bottom and the top free surface. The reflective conditions ensure a zero particle flux through the walls. The strategy of the simulation is to move, at each time step δt, the particle upward or downward by an elementary step $\delta y_i = \sqrt{2 D_i \delta t}$, which depends on the layer Li where the particle stands.

We have reported, in Fig. 9(a), the results from the simulation concerning the mean particle displacement according to its initial position. One can first note that particles from intermediate layers (L2 to L6) have a net upward motion with a constant drift velocity of 0.09 cm/s. This is a classical result of inhomogeneous diffusion: indeed, it can be shown that the mean displacement of a particle diffusing in an infinite medium, characterized by a spatial-dependent diffusion coefficient $D(y)$, is nonzero and the mean drift velocity scales as $\partial_t \delta \sim \partial_y D(y)$. In the particular case where $D(y)$ has a linear dependence on y, the drift velocity is independent of the particle position. Second, it is clear from Fig. 9(a) that the presence of the reflective walls plays an important role in the transverse motion of the particles from the lower and upper layers. The mean displacement of particles from the layer L1 is enhanced in comparison with that of bulk particles (the drift is found to be 0.4 cm/s), whereas the particles from the top layer L8 have a net important downward motion (corresponding to a negative drift velocity of 1.2 cm/s). One should note that the presence of the reflective walls has a more dramatic effect on the upper layer where the diffusion coefficient is greater.

It is now interesting to compare the mean transverse motion of the flow particles obtained from our basic simulation with the experimental data [cf. Fig. 9(b)]. One clearly sees that the simulation outcomes differ quantitatively from those obtained in experiments. Indeed, the main discrepancy is that the simulations greatly underestimate the amplitude of the mean displacement for the bulk particles. As a consequence, the magnitude of the upward migration of the bulk particles cannot be solely explained by the presence of the gradient in the mass diffusion coefficient. We can evoke at least three effects which could favor the upward migration of the particles. First, we have assumed a constant packing fraction in the flow, as suggested by the experimental measurements. However, a careful inspection of the packing fraction profile shows that it slightly decreases as one moves away from the base of the flow. This slight variation of the packing fraction could induce upward migration of the particles. Second, the local fluctuations of the packing fraction in a given layer should also be a crucial parameter for particle migration. Indeed, the transfer of particles from one layer to an adjacent one should depend, among other things, on the probability of finding a void space in the latter, and one can reasonably expect that this probability is higher for the upper layers (since the granular temperature is greater), resulting in a favored upward migration. Third, the boundary conditions could play an important role. The top layer is a free surface whereas the bottom layer is bounded by the rough base. This asymmetry could eventually also favor an upward migration.

III. BIDISPERSE FLOWS

A. Motivation

It is well known that when different particles are put into relative motion by shearing, flow, or vibration, they do not mix and show a tendency to gather according to their features. This segregation process occurs when particles exhibit differences in size [19–23], density, shape, or roughness [24]. For particles of the same composition, the segregation of small and large beads has been investigated in numerous studies, and generally small particles are found at the bottom surface of a medium composed of large particles (or, conversely, large particles are located at the top surface of a medium composed of small particles). This segregation mechanism has been explained by geometrical effects: as particles flow, it is more likely for a small particle to find a large enough void space to fall into than for a large particle.

On this basis, Savage and Lun [19] proposed a theoretical model for the segregation process in dense granular flow. They argue that two mechanisms compete for the transfer of particles between layers in relative motion: (i) a random fluctuating sieve process which is size dependent and induced by gravity, and (ii) a mechanism of squeeze expulsion due to imbalances in contact forces on an individual particle (this latter mechanism has no size nor direction preferences). The combination of these two mechanisms leads to a net percolation velocity of each species and, by means of the mass conservation equation, the development of the concentration profile with downstream distance can be obtained. Concerning the concentration profile and the distance required for the complete separation of particles, reasonable agreement is found between the predictions of the model and the measurements from 3D flows of a binary mixture down a rough inclined chute [19,25].

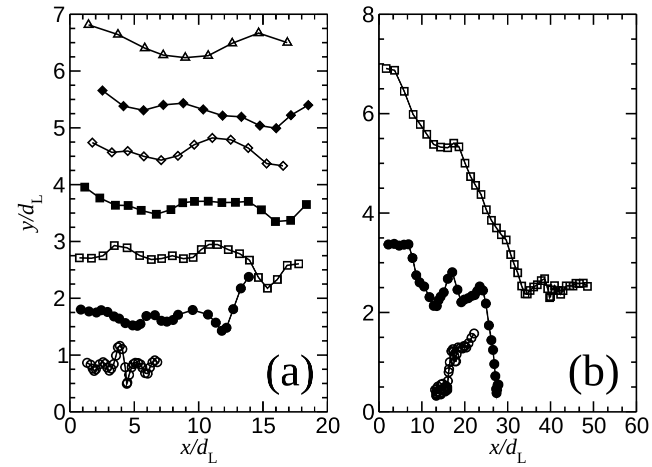

FIG. 10. Selected trajectories of (a) large disks and (b) small ones in bidisperse chute flows. Each symbol corresponds to the center of gravity of a different disk.

However, only a few experiments have been carried out on the segregation process itself during dense inclined chute flows of granular materials. Our objective is to study this process carefully through our 2D inclined chute flows consisting of binary mixtures of coarse and fine particles and to test the model of Savage and Lun [19].

B. General properties of the bidisperse flow

The flow experiments were performed with binary mixtures using the same inclined chute as that described in Sec. II. The binary mixtures were made of large polystyrene disks of diameter $d_L=8$ mm and small ones of diameter $d_S=4$ mm, giving a diameter ratio of 2. We worked only with very dilute mixtures (e.g., less than 1% of fine particles). The experimental conditions are the same as those used for monodisperse flows: a chute inclination of 36° and a flow height of eight large disk diameters. We observe a quasi-fully-developed flow 1 m downstream from the beginning of the chute which lasts about 2 s. Strictly speaking, the flow is not fully developed since through the chute the segregation process occurs: the small disks migrate progressively toward the lower layer of the flow. However, as the mixture is dilute, the flow of large disks remains stationary and homogeneous along the longitudinal direction.

In Fig. 10, we present some characteristic trajectories for both large and small disks. The time interval between two consecutive points on each particle trajectory corresponds to $\tau_0=20$ ms. Since the binary mixture is very dilute the large disks within polydisperse flows are found to exhibit the same behavior as in monodisperse flows. One can note that the large particles travel roughly at a constant height with some fluctuations (smaller than one disk diameter).

In contrast, the small particles exhibit an important mean transverse motion but fluctuations are still present. Some recorded trajectories of small disks show a clear net downward motion. This is a manifestation of the segregation process and according to Savage and Lun [19] it can be explained as follows. Because of the overriding of layers and the con-

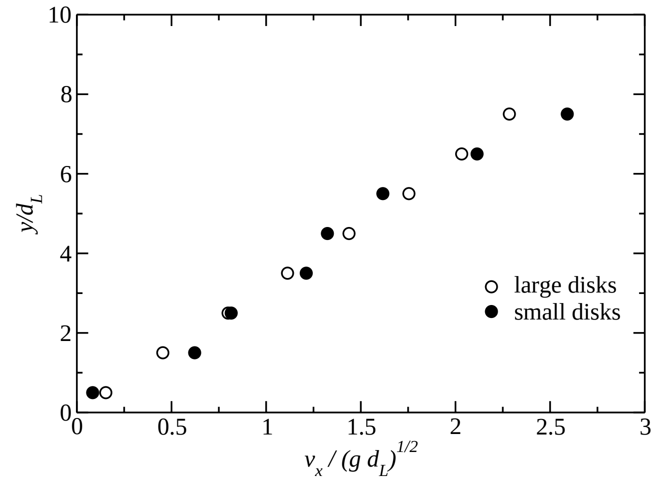

FIG. 11. Mean longitudinal velocity V_x as a function of the height from the bottom for small and large disks.

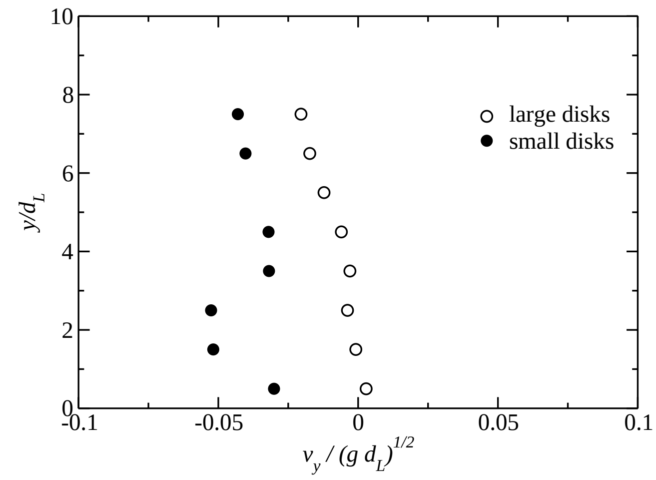

FIG. 12. Mean transverse velocity V_y as a function of the height from the bottom for small and large disks.

tinual rearrangement of particles within a layer, the contact force network and the void spaces are undergoing continual random changes. At any instant, there is a distribution of void spaces in a given layer. If a void space is large enough, then a particle from the layer above can fall into it as the layers move relative to one another. The probability of finding a hole into which a small particle can fall is larger than that of finding a hole into which a large particle can fall. Hence, there is a tendency for the particles to segregate out, with fine ones at the bottom and coarse ones at the top. This is the gravity-induced void filling mechanism proposed by Savage and Lun [19]. However, it should be noted that small particles from the bottom have a net upward motion which is clearly caused by the presence of the boundary.

We measured the mean velocity of the small particles in the flow and compared it to the mean velocity of large disks. We have plotted in Fig. 11 the mean longitudinal velocity V_x of each species as a function of the height measured from the bottom. There is no difference between small and large disks concerning the mean longitudinal velocities. The flow shear rate γ is 10 s^{-1}. In contrast, the mean transverse velocity profile (see Fig. 12) is different for small and large particles. Whereas the mean transverse velocity of large particles is almost equal to zero, small particles have a significant downward velocity. It should be noted that for small disks the averages were performed over a particle population whose repartition across the flow is not uniform (see Fig. 13). In view of the experimental data, it is difficult to say whether the downward migration speed is independent of the height y or not. The migration speed averaged over the height of the flow is found to be 1.2 cm/s.

The mechanism responsible for the net downward migration has been described by Savage and Lun [19] and has been termed a ‘‘random fluctuating sieve.’’ This phenomenon is a gravity-induced, size-dependent, void filling mechanism. Voids are characterized by a ‘‘void disk’’ defined as the largest grain that can fit into a given void space. The authors derived a theoretical expression for the downward percolation velocity V_d which is given in the limit of a dilute mixture by

$$V_d/\gamma d_l = \frac{4}{\pi}\frac{M}{N}\nu\left[(2+\bar{E}-E_m)\exp\left(-\frac{1-E_m}{\bar{E}-E_m}\right) - (\sigma+\bar{E}-E_m+1)\exp\left(-\frac{\sigma-E_m}{\bar{E}-E_m}\right)\right], \qquad (5)$$

where M/N is the ratio of the number of voids to the number of particles in a given layer, $\bar{E}$ is the mean void diameter ratio defined as the ratio of the mean void diameter to the mean diameter particle (which corresponds to the large disk diameter in the dilute mixture limit), $\sigma = d_S/d_L$ is the ratio of the small disk diameter to the large one, and E_m is the minimum possible void diameter. For close packing of equal disks, $E_m = 0.1547$. As our mixture is very dilute, it is legitimate to take this value for E_m. Finally, ν is the volume fraction of the flow. All these parameters can be estimated

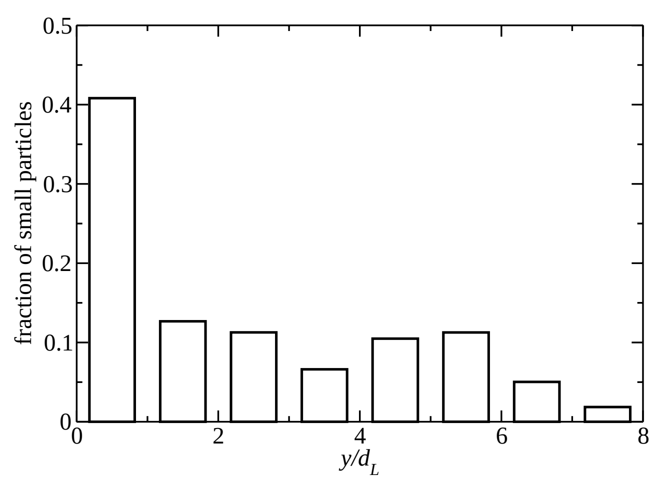

FIG. 13. Repartition of the small particles in the flow as a function of the height y within the observation window.

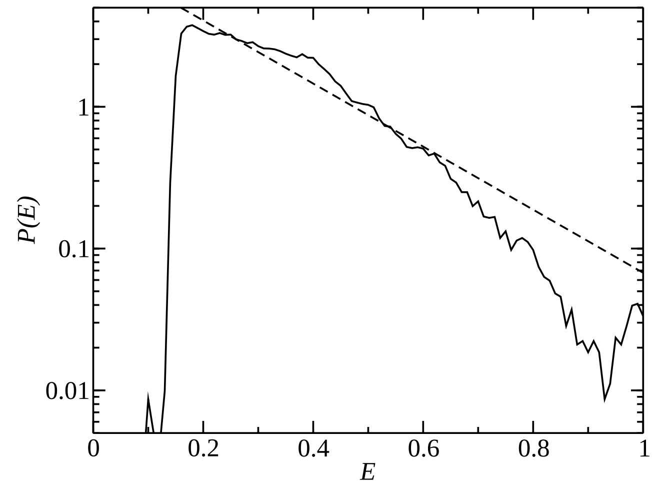

FIG. 14. Experimental pore size distribution within the flow (continuous line). The dashed line represents the theoretical distribution calculated for $\overline{E}=0.35$ (which corresponds to the value extracted from the experimental data) and $E_m=0.1547$.

from an analysis of the video recording of our flow. In order to determine the features of the voids in the flow, we employed a method very similar to that of Savage and Lun [19]: we mapped the interparticle space onto the Voronoï network [26] and defined a void disk as the largest disk that can fit into a Voronoï vertex [27]. We found $M/N \approx 0.6$ and $\overline{E} \approx 0.35$. In addition, the packing fraction ν was found to be of order 0.7. Using these parameters, we obtained $V_d = 0.3\gamma d_L \approx 1.6$ cm/s. This value has to be compared to the experimental measured percolation speed $V_d \approx 1.2$ cm/s. As a conclusion, the theoretical expression for the percolation speed gives a rather good estimation.

The theory of Savage and Lun is based on the assumption that the probability of finding a voids diameter ratio $E = d_v/\overline{d}$ is given by a decreasing exponential law of E:

$$p(E) = \frac{1}{\overline{E}-E_m}\exp\left(-\frac{E-E_m}{\overline{E}-E_m}\right). \qquad (6)$$

This distribution function can be derived in the framework of the "maximum entropy approach." It would be interesting to test the pertinence of this theoretical pore size distribution function experimentally. The experimental estimation of this distribution function is shown in Fig. 14. It is found that the tail of $p(E)$ (i.e., for $E>0.4$) exhibits a clear exponential decay but with a decay rate greater than that calculated from the theory. This result tends to show that Eq. (6) can be used only as an approximation to zero order. There is therefore a need to improve the theoretical description for a better agreement with the experiments.

C. Segregation and transverse displacements

Examination of the transverse displacements of the small particles gives us some insights into the segregation process. The mean displacements of the small disks are shown in Fig. 15(a). One can note again that the small disks have a net downward motion, except for those which are located in the bottom layer. The mean displacement of the particles varies roughly linearly with time. Furthermore, the particles from the upper layers exhibit a greater downward displacement than those from the lower layers.

From the mean transverse displacement, one can extract a migration speed, which will be denoted V_d^*. It is important to realize that V_d^* is different from the percolation speed V_d calculated in the previous subsection. V_d is a mean local velocity which is obtained by averaging the velocity of the particles belonging to a given layer and is the analog of a Eulerian velocity (or a fluidlike velocity). On the contrary, V_d^* is determined from the individual motion of the particles which are initially located in a given layer and will be referred to as the "particle velocity." V_d^* is calculated from the slope of the curves plotted in Fig. 15(a) and is shown in Fig. 16. One can note than V_d^* increases roughly linearly with the transverse position in the flow and differs notably from the Eulerian percolation speed V_d. At this point, one should make a comment. It is not really surprising that the Eulerian velocity and the particle velocity differ. Even in a classical fluid, the velocity of the molecules is not equivalent to the mean fluid velocity (i.e., the Eulerian velocity) [28].

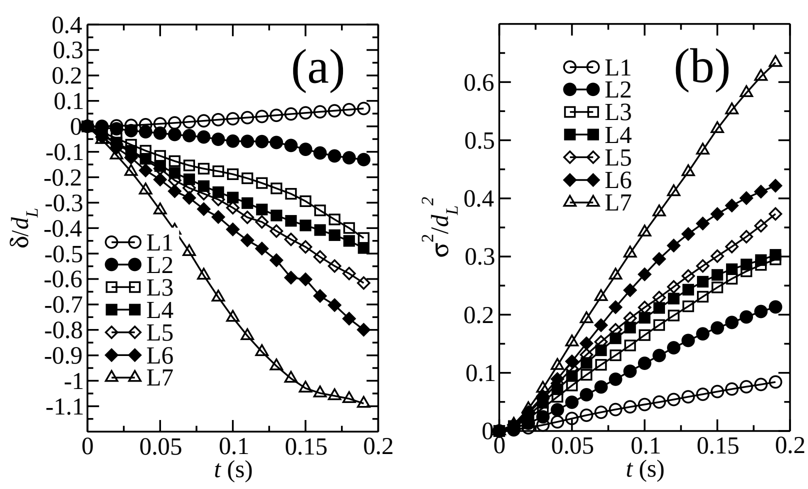

FIG. 15. (a) Mean transverse displacements and (b) variance of vertical displacements as a function of time according to the particle position.

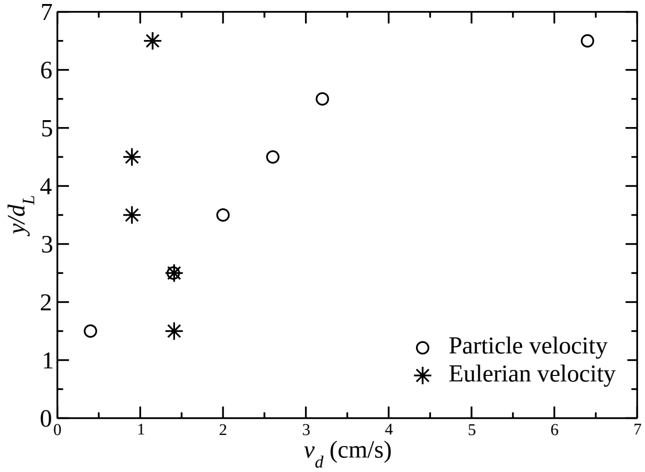

FIG. 16. "Particle" percolation speed V_d^* (○) and "Eulerian" speed $V_d = -V_y$ (*) as functions of the height from the flow base.

We also calculated the fluctuations of the transverse particle displacements according to their initial position in the flow [see Fig. 15(b)]. We observe a behavior very similar to that of the large particles. The variance of the particle displacement increases linearly with time. The slope depends on the transverse particle position: it increases when the particle is higher in the flow. The small particles therefore have a diffusive motion superimposed on a net downward motion. This diffusion process can be associated with the "squeeze expulsion mechanism" described by Savage and Lun. It has no preferential direction of layer transfer and is size independent, as observed for the diffusion process (see Fig. 17). We have indeed compared the diffusion coefficient associated with the small particles to that associated with the large ones. The diffusion behavior seems independent of the particle size. For both particle sizes, the diffusion coefficient is of the same order of magnitude and is an increasing function of the layer height.

To quantify the strength of the diffusion process relative to the segregation one, we have evaluated the ratio between

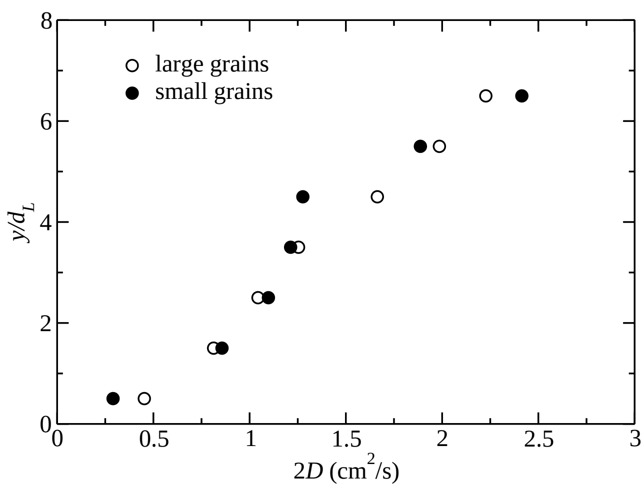

FIG. 17. Diffusion coefficient as a function of the height from the flow base for large and small disks.

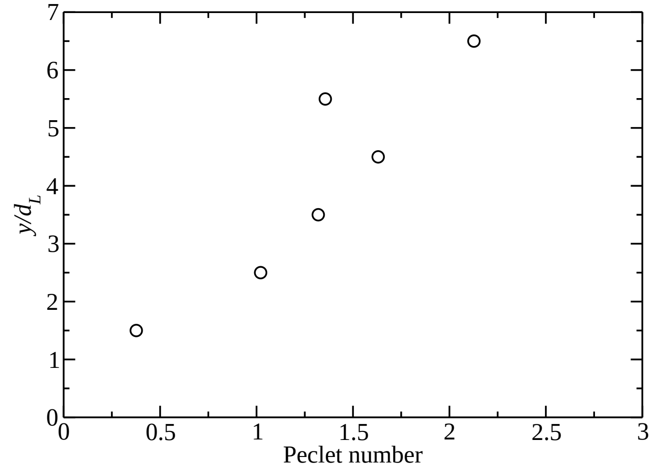

FIG. 18. Ratio between the diffusive characteristic time τ_D and the convective one τ_C as a function of the height y from the flow base.

the characteristic diffusive time τ_D and the characteristic convective one τ_C (associated with the net downward motion). τ_C is defined as the time needed for a particle to change layer due to the convective motion: $\tau_C = d_L / V_d^*$. τ_D is associated with the diffusive motion and is given by $\tau_D = d_L^2/2D$. The ratio $\tau_D/\tau_C = d_L V_d^*/2D$, analogous to a Péclet number, is plotted in Fig. 18 as a function of the particle position in the flow. We can note that this ratio increases as one moves away from the flow base, indicating a stronger segregation process in the high part of the flow. However, it remains of the order of unity which means that there is not a strong predominance of one process over the other.

D. Discussion

We would like to discuss here the observed difference between Eulerian and particle percolation speed. In particular, we want to know whether the layered structure of the flow (with a nonhomogeneous mass diffusion coefficient) is responsible for that difference. To answer this question, we simulated the transverse individual movement of a small particle by a one-dimensional biased random walk along the y direction. The particle moves in a layered medium where the diffusion coefficient varies from one layer to another, as observed in the experiments. A bias V_d^* is added to mimic the downward convective motion of the small particles. As suggested by the experimental data, this bias is taken to be dependent on the initial vertical position of the particle in the flow. We have chosen a linear variation with the height y: $V_d^* \approx a(y/d_L) + b$, with $a = 0.66$ cm/s and $b = -0.83$ cm/s. The strategy of simulation is the same as that described in Sec. II.

In Fig. 19, we have presented the mean displacements of the particles obtained for different initial particle positions. For particles in the bulk, the mean displacement varies linearly with time and is directly related to the particle migration speed V_d^* introduced in the simulation: $\partial\delta/\partial t \approx V_d^*$. We calculated the corresponding Eulerian percolation speed, considering an initially uniform distribution of small par-

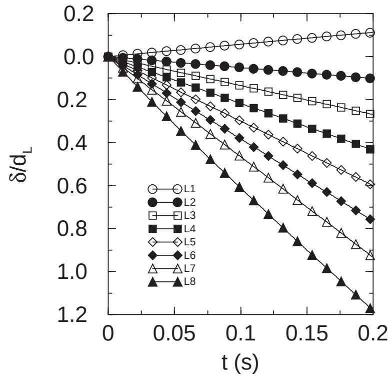

FIG. 19. Results from the simulation: mean vertical displacements as a function of time for different initial particle positions.

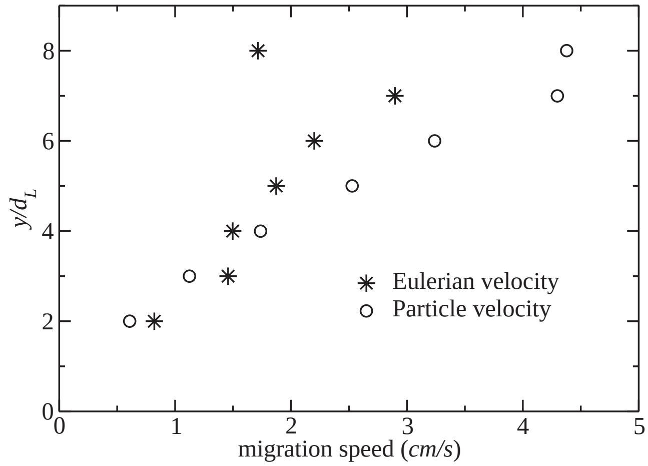

FIG. 21. Particle and Eulerian downward migration speeds, calculated from the simulations based on a modified random walk (see text), as a function of the vertical position in the flow. The simulation was achieved considering an initially heterogeneous distribution of particles identical to that found experimentally.

ticles in the system and letting them evolve for 0.2 s. The results are shown in Fig. 20. We find that the Eulerian percolation speed behaves like the particle one and is slightly greater. The latter result is not surprising since the Eulerian speed in a given layer is the average of particle velocities of particles belonging to that layer and particles coming from the neighboring layers. In particular, particles coming from the neighboring upper layer have a greater particle speed and therefore contribute to enhancing the Eulerian velocity from that layer. However, this is not what is experimentally observed since the Eulerian speed is smaller than the particle one, especially for the upper layers of the flow.

In conclusion, this simple model based on a classical biased random walk in a stratified medium is not able to reproduce the difference observed between the Eulerian and particle percolation speeds. Several hypotheses can be proposed to explain this difference. (i) First, the heterogenous distribution of small particles in the flow along the transverse direction (cf. Fig. 13) may change the estimation of the Eulerian speed. (ii) Second, it may be possible that the particle motion is not as simple as a classical random walk. A careful

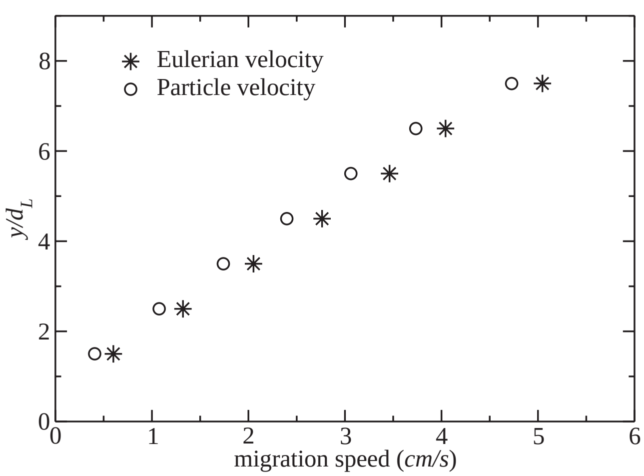

FIG. 20. Particle and Eulerian downward migration speeds, calculated from simulations based on a classical Brownian random walk, as functions of the vertical position in the flow. The simulation was achieved considering an initially uniform distribution of particles in the system.

observation of the reported trajectories of the small particles (cf. Fig. 10) suggests the existence of two different spatial scales: one associated with the diffusion process within each layer and the other related to the downward migration process. Indeed, when a particle from a given layer leaves its layer, it travels in a straight path over a distance which can be several layer width. This fluctuating behavior bears some resemblance to a Lévy flight. (iii) Third, the motion of the particles in the flow direction is not taken into account in the simulation. In particular, particles from the upper layers move faster and cross the observation window in a shorter time than that needed for the particles from the lower layers. As a result, upper particles contribute to the statistics with a smaller weight.

We have tested the two first hypotheses. The introduction of a heterogeneous distribution of the small particles along the transverse direction of the flow in the simulation does not quantitatively change the results. On the contrary, if we simulate a random walk introducing two spatial scales (one for the diffusion process and the other for the migration mechanism) as evoked above, combined with a heterogenous distribution of particles (chosen identical to the experimental one), we find a much better agreement with the experiment. The rules used for the random walk are the following: a particle from a given layer undergoes at short time a random Brownian motion with a diffusion coefficient corresponding to that measured experimentally, and as soon as the particle leaves its own layer downward by this diffusion process, it is animated by a persistent motion in the downward direction over a distance of one-half layer width. Then the particle diffuses again in its new layer, and so on. The results of this simulation are shown in Fig. 21. In the upper layer, the Eulerian percolation speed is found to be smaller than the particle speed as in the experiment in the upper layers. In the case of a random walk based on two spatial scales, the Eulerian percolation speed is very sensitive to the initial distribution of the small particles in the flow, whereas the particle speed is rather independent. These results are encouraging, and it therefore seems important in the near future to study the particle trajectories in more detail.

IV. CONCLUSION

In the first part, we showed the existence of a transverse diffusive particle motion in a fully developed granular flow of monosized particles on an inclined chute. Particles from a given layer have a nonzero probability of being transferred to adjacent layers. We found that the mean residence time in each layer is finite and depends on the layer height in the flow. The diffusion coefficient associated with this diffusive motion is an increasing function of the layer height. In addition, particles from the bulk have a tendency to migrate upward, which can be partially explained by the presence of the diffusion coefficient gradient.

In the second part, we reported experimental results for the segregation process in 2D dense inclined chute flows of binary granular mixtures. At short time scales, we showed the existence of two competing processes driving the motion of the small particles diluted among large particles: a transverse downward convective motion, which is a gravity-induced, size-dependent, void filling mechanism as described by Savage and Lun [19], and a diffusive process, which is not size preferential and has no preferential direction for the layer transfer. For a diameter ratio of small particles to large ones equal to 1/2, we found that the convective and diffusive processes manifest themselves on the same time scale, which means that they are of the same strength. Therefore the segregation mechanism is only efficient at long time scales.

We have also identified two different percolation speeds: (i) a local one equivalent to a Eulerian velocity and (ii) a particle speed associated with the individual particle motion.

(i) The Eulerian percolation speed is found to be roughly independent of the transverse position of the particles and has been compared to the prediction of the Savage and Lun theory. We found a qualitative, but not fully satisfactory, agreement. The proposed theory is based on information-entropy concepts for the description of voids in the flow. The validity of the applicability of these entropy concepts may be questioned. A definite answer to this issue could be obtained through careful experimental measurements of the distribution of voids in the system. This can, in principle, be done with our experimental 2D chute flow setup where all the features of the flow can be analyzed via a high speed camera.

(ii) The particle percolation speed is different from the Eulerian one and, in particular, is greater for particles from the upper layers than from the lower ones. We propose some possible explanations for the latter observation. An appealing hypothesis is that the particle follows a more complex random walk than a simple biased Brownian motion. In particular, a random walk based on two different spatial scales, one associated with a diffusing process within each layer and the other attached to the downward migration, reproduces the experimental results with a rather good qualitative agreement. This result tends to show that the segregation process is more complex than a classical biased diffusion process. Other hypotheses should be investigated as to the importance of the relative motion between layers due to the velocity gradient in the transverse direction of the flow.

As a conclusion, the segregation process in dense flows is far from being comprehensively understood, and additional experimental and theoretical efforts are strongly needed. In particular, our next objective is to fully characterize the particle motion, which is feasible with our 2D experimental setup.

ACKNOWLEDGMENTS

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